

Renormalization Flow, Duality, and Supersymmetry Breaking in Some $N = 1$ Product-Group Theories

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We discuss the renormalization group flow, duality, and supersymmetry breaking in $N = 1$ supersymmetric $SU(N) \times SU(M)$ gauge theories.

1. Motivation

Duality, which relates the strongly coupled behavior of one gauge theory to the weakly coupled behavior of another, has emerged as a key idea in the understanding of the nonperturbative dynamics of supersymmetric gauge theories. Most of the work in this context has focused on gauge theories with simple gauge groups. One would like to understand theories with non-simple groups in more detail:

- i. Such an investigation will serve as a non-trivial check of simple-group duality. Gauging a global symmetry in two theories related by Seiberg duality is often a relevant perturbation, and the equivalence of the resulting two theories will give further evidence for duality.
- ii. Product groups often arise in the course of dualizing theories with simple groups, once one goes beyond the simplest matter representations.
- iii. Many phenomenologically interesting chiral gauge theories have product gauge groups.
- iv. Several classes of product-group theories exhibit dynamical supersymmetry breaking.

2. The Renormalization Flows

An important question that arises in product-group theories is whether the infrared physics changes when one varies the ratio of the strong coupling scales of the two groups. A clue to the answer is given by studying the renormalization flow in the space of the two gauge couplings. We

will consider a theory based on the gauge group $SU(N) \times SU(M)$ with a single (\square, \square) field, and a number of additional (anti)fundamentals of each group. The matter content is thus completely specified by the number of flavors, N_f , of $SU(N)$ in the limit when the $SU(M)$ gauge coupling is turned off, and the number of flavors, M_f , of $SU(M)$ when $SU(N)$ is turned off.

To analyze the flows, we assume that M, N, M_f, N_f are such that in the absence of the other gauge coupling, each gauge theory flows to an infrared fixed point. By turning off, say, the $SU(M)$ gauge coupling g_M , the $SU(N)$ gauge coupling g_N flows to the Seiberg fixed point. Upon weakly turning on g_M , the theory flows arbitrarily close to the g_N fixed point, in the vicinity of which the anomalous dimensions of all matter fields are approximately known (only the large anomalous dimensions of the $SU(N)$ fundamentals at the fixed point, determined by their R charge, are important in our analysis). Using the relation between the beta function and anomalous dimensions, $\beta_{g^2} \sim -g^4 (3T(G) - \sum_R T(R) (1 - \gamma_R))$, we can find whether gauging $SU(M)$ is relevant at the $SU(N)$ fixed point. Repeating the analysis in the vicinity of the $SU(M)$ fixed point, we obtain a set of inequalities [1], depending on M, N, M_f, N_f , which determine when gauging the flavor subgroup is a relevant perturbation at each of the fixed points.

The three possibilities—up to interchanging the gauge groups—are presented on fig.1. It is worth noting that only the flows from figs.1a. or 1b. occur for the allowed values of M, N, M_f, N_f . The flow from fig.1c. would imply the existence

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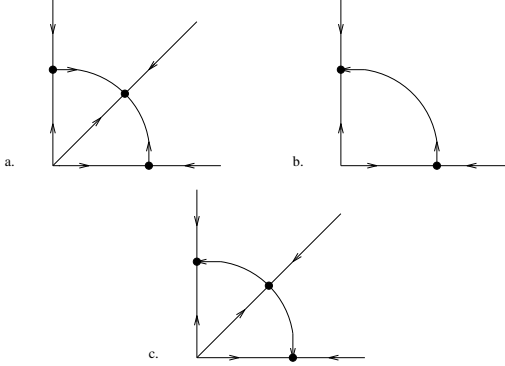


Figure 1.

of a phase transition, as the ratio of the two gauge couplings is varied. There are arguments, based on holomorphy, against the existence of phase boundaries in supersymmetric theories. We note, however, that the analysis of the flows above involves the anomalous dimensions, which are not holomorphic functions in $N = 1$ theories. Nevertheless the analysis is locally—in the vicinity of the two fixed points—consistent with the absence of a phase transition. The global nature of the flows can be studied in the weak coupling (Banks-Zaks) limit; the analysis also yields only the flows of figs.1a,b. [1].

3. Product-Group Duality

We next turn to studying duality in product-group theories. We analyze in detail the case of $SU(2)_1 \times SU(2)_2$ with a single $(2, 2)$ field and additional n flavors for $SU(2)_1$ and m flavors for $SU(2)_2$. The scales of the two groups are Λ_1 and Λ_2 ; we call these theories the “[n, m] models”.

Using Seiberg duality, we can construct many duals of a given product-group theory. Motivated, say, by $\Lambda_1 \gg \Lambda_2$, we can use Seiberg duality of $SU(2)_1$ to construct an $SP(2n-4)$ dual. Weakly gauging the $SU(2)_2$ flavor symmetry, we obtain what we call the *first dual*, an $SP(2n-4) \times SU(2)_2$ gauge theory. Using Seiberg duality for $SU(2)_2$, we can now construct another dual theory, the *second dual* $SP(2n-4) \times SP(4n+2m-10)$

theory. This process can be continued by further dualizing the first group in the second dual, etc., however, the SP now has an antisymmetric tensor, and its dual is still unknown. As an aside, we note that in the general $SU(N) \times SU(M)$ theories—where this process can be continued by using only SQCD Seiberg duality—the chain of duals closes, so there is only a finite number of duals that can be constructed by using simple-group duality. Returning to the [n, m] models, in the opposite, $\Lambda_1 \ll \Lambda_2$, limit, we can construct other first and second duals, which have the above groups with m and n interchanged.

Our purpose is to show that all these duals of the original [n, m] models are equivalent and independent of the ratio of couplings, the smallness of which was used to motivate the construction of the duals. To this end, we perform the following detailed checks [1]:

1. The anomaly matching conditions are automatically satisfied, since at each step we use Seiberg duality.

2. Consistency of duality with mass perturbations: we show that all duals, constructed above, flow under mass perturbations in a way consistent with duality, and that the scale matching conditions for both gauge groups also change consistently with the flows.

3. Consistency of duality with deformations along flat directions: we show that the moduli spaces of all duals are identical to that of the electric theory. Classical restrictions on the moduli space in the electric theory often arise from nonperturbative effects in the duals. In the case of product groups, this involves nontrivial interplay of the nonperturbative effects in both dual gauge groups. We show that the chiral rings of all duals are the same.

4. Mass flows to the confining phase: we show that the duals of the [$2, m$] models flow to the confining phase upon adding appropriate mass perturbations. Duality predicts that the confining superpotentials arise through—as yet poorly understood—nonperturbative effects involving instantons in both the broken and unbroken factors in the dual.

5. We study in detail the moduli space of the “partially confining” models where one of the

electric gauge groups would confine if the other were turned off. We show that the dual theories reproduce the moduli space of the electric theory, including the quantum deformation of the moduli space in the $[1, m]$ models. The $[1, m]$ models exhibit another nonperturbative effect, specific to product-group theories: the dynamical generation of a dilaton-axion superfield required by Green-Schwarz anomaly cancellation. We show how duality helps determine the dilaton field.

6. We also study the confining phase and find the exact superpotentials. The infrared physics in the confining phase is also independent of the ratio of couplings.

7. We note that adding Yukawa perturbations of appropriate rank, without any mass terms, can also drive the theory into the confining phase. In the first dual, the Yukawa terms are mapped into mass terms that reduce the number of flavors. In the second dual, the Yukawas completely higgs one of the dual gauge groups, and (incompletely) higgs and reduce the number of flavors of the other sufficiently to make it confining. For smaller rank Yukawa couplings, duality predicts the existence of new nontrivial fixed points.

While we have performed the detailed checks for only $M = N = 2$, we expect that our results are valid more generally, and that the obtained insight will be useful in analyzing more complicated product-group theories.

4. Supersymmetry Breaking

We show that a large class of the $SU(N) \times SU(M)$ models break supersymmetry: here we will only briefly consider the “partially confining” models with $N_f = M$ and $M_f = N$ ($N > M$). The models with $M = N - 1$ are considered in ref. [1], and the smaller- M ones in [2]; the latter are interesting since $SU(M)$ is not confining when $SU(N)$ is turned off. We show that upon adding a maximal rank Yukawa coupling, these models break supersymmetry. Although they possess classically flat directions, these are lifted in the quantum theory, and the models may have stable supersymmetry-breaking vacua (one can also add higher dimensional operators to lift the flat directions, without restoring supersymmetry).

5. Conclusions:

1. Gauging a flavor subgroup is often a relevant perturbation in a simple-group theory. The consistency of the flows in the electric and magnetic theories under this perturbation provides yet another consistency check on Seiberg duality.
2. Our results imply that, quite generally, duals of product groups can be constructed by using known dualities for the simple groups in the products.
3. The study of the renormalization flows and the detailed checks on duality indicate that the infrared physics is independent of the ratio of the strong coupling scales of the two gauge groups.
4. We showed that it is possible to flow to the confining phase by dimension-3 perturbations only. This raises the possibility of constructing new phenomenologically interesting models of dynamical supersymmetry breaking.
5. We found new interesting nonperturbative phenomena specific to product groups, such as the dynamically generated dilaton in the “partially confining” models.
6. We showed that a large subset of the $SU(N) \times SU(M)$ models break supersymmetry after adding suitable dimension-3 perturbations.

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